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MULTI-ATTRIBUTE UTILITY THEORY: A PSYCHOLOGICAL PERSPECTIVE

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Multi-attribute utility theory is discussed from a psychological point of view. The various independence assumptions underlying the riskless additive utility model, the additive expected utility model, the multiplicative expected utility model, and the quasi-additive expected utility model are formulated and illustrated. Empirical studies which shed light on the descriptive validity of these axioms are reviewed and some suggestions are made for future psychological research on multi-attribute utility theory.

The term multi-attribute utility theory (MAUT) refers to a set of models that are meant either to 1. describe how subjects make decisions that involve multidimensional outcomes or 2. prescribe how subjects ought to make such decisions. For example, from a descriptive point of view we are interested in whether existing multi-attribute utility models adequately describe how a person chooses between cars varying in price, fuel economy, and size. On the other hand MAUT can be used to specify which car a subject should buy in order to be rational, given the value and importance he or she attaches to each attribute.

As a normative theory MAUT is often considered as a decision aid that helps people to make judgments satisfying certain intuitive notions of rationality, such as, for example, transitivity. For an extensive survey of MAUT from a prescriptive point of view, the reader is referred to Keeney and Raiffa (1976).

Although characterizing people's intuitive ideas about rationality is an interesting topic in its own right, it will not be pursued in this paper. I shall rather concentrate on specifying the empirical conditions a decision maker (DM)'s judgments must satisfy, in order for them to be representable by the various multi-attribute utility models. These empirical conditions form a subset of the axioms on which the representation theorems for the models are based. When a DM's judgments are found to satisfy the axioms, they can be represented numerically according to the model being tested. Although the functional form of the model can provide some insight into the psychological processes

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underlying the judgments, it is my conviction that investigating the conditions under which specific axioms hold reveals far more about the psychological processes involved than do the models themselves.

When studying choices among multi-attribute alternatives, a distinction is usually drawn between decisions made under certainty, so-called riskless choices, and decisions made under uncertainty, so-called risky choices. Whereas in the former situation the DM is sure about the outcome of his or her choice, this is not the case for risky choices.

Some notation needs to be introduced in order to state the various independence conditions implied by the models representing riskless and risky choices. Unless otherwise stated, it is assumed that at least three attributes A_i ($i = 1, n$) are involved, where n is the number of attributes. Moreover it is assumed that for $n \geqslant 3$, at least three attributes are essential in the sense of Krantz, Luce, Suppes, and Tversky (1971, p. 256). Loosely speaking, this means that at least three attributes influence the choices made by the DM. The Cartesian product of all attributes, which is denoted as A, is called the outcome space: $A = \times_{i \in N} A_i$, with N defined as $\{1, 2, \ldots, n\}$. An outcome $\mathbf{x} \in A$ is an *n-tuple* $(x_1, x_2, \ldots x_n)$, where x_i is the level of $\mathbf{x}$ on A_i. For riskless decision problems, A constitutes the set of alternatives and $\dot{\geqslant}$ will be used to denote an observed preference relation on A. For all numerical representations considered in this paper, $\dot{\geqslant}$ is required to be a weak order (i.e., transitive and connected).

In risky situations, choices are not made among outcomes, but among probability distributions defined over the outcome space. Consequently in this case the set of alternatives equals π, the set of all possible probability distributions over A, and $\dot{\geqslant}$ is defined over π. Since π only consists of simple probability distributions, the probabilities assigned to the outcomes by each $P \in \pi$, sum up to one. π is often referred to as the decision space. Following Farquhar (1977, 1978) we define for $I \subset N$ and $I \neq \varnothing$, $\bar{I} = N - I$, $A_I = \times_{i \in I} A$, and $A_{\bar{I}} = \times_{i \in \bar{I}} A_i$. Note that $A = A_I \times A_{\bar{I}}$. Similarly to π, π_I denotes the set of all simple probability distributions over A_I. For a fixed $P_I \in \pi_I$, and a fixed $\mathbf{a}_{\bar{I}} \in A_{\bar{I}}$, $(P_I, \mathbf{a}_{\bar{I}})$ denotes that alternative in π which has marginal probability P_I on A_I and assigns probability one to $\mathbf{a}_{\bar{I}} \in A_{\bar{I}}$.

DECISIONS UNDER CERTAINTY

According to MAUT, people in riskless choice situations select the alternative with the highest subjective utility. This means that $\mathbf{x}$ is preferred to $\mathbf{y}$ whenever its utility exceeds that of $\mathbf{y}$, or formally

$$\mathbf{x} \dot{\geqslant} \mathbf{y} \text{ iff } U(\mathbf{x}) \geqslant U(\mathbf{y}) \tag{1}$$

where U is a real-valued utility function on A. Transitivity is the fundamental property the empirical relational system $(A, \dot{\geqslant})$ must satisfy in order to be representable according to (1). Tversky (1969) has convincingly demonstrated that transitivity is not a trivial property of choices among multidimensional alternatives.

Provided certain independence assumptions are satisfied, $U(\mathbf{x})$ can be decomposed into independent components, each corresponding to a separate attribute. MAUT favors the additive model

$$U(\mathbf{x}) = \sum_{i \in N} u_i(x_i) \tag{2}$$

where u_i is a real-valued function on A_i. Model (2) in connection with (1) is referred to as the riskless additive utility model. It is based on the assumption, often made in consumer economics, that the utility of a commodity bundle equals the sum of the utilities of the individual items. In order to state the additional restrictions the additive model imposes on $(A, \dot{\succcurlyeq})$, we need to introduce the concept of preference independence (sometimes called weak conditional utility independence).

DEFINITION 1. A_I is *preference independent* of $A_{\bar{I}}$ iff, for all $\mathbf{x}_I, \mathbf{y}_I \in A_I$, $(\mathbf{x}_I,\mathbf{a}_{\bar{I}}) \dot{\succcurlyeq} (\mathbf{y}_I,\mathbf{a}_{\bar{I}})$ for some $\mathbf{a}_{\bar{I}} \in A_{\bar{I}}$, implies $(\mathbf{x}_I,\mathbf{b}_{\bar{I}}) \dot{\succcurlyeq} (\mathbf{y}_I,\mathbf{b}_{\bar{I}})$ for every $\mathbf{b}_{\bar{I}} \in A_{\bar{I}}$.

If the index set I consists of only one or two elements, preference independence is equivalent to the well-known conjoint measurement axioms of respectively single factor independence and joint independence (cf. Krantz et al., 1971). Whenever $(A, \dot{\succcurlyeq})$ is a weak order and A_I is preference independent of $A_{\bar{I}}$, for every $I \subset N$, $I \neq \varnothing$, real-valued functions u_i on A_i exist which satisfy (1)-(2), given certain additional structural assumptions. Moreover, the utility functions are unique up to positive linear transformations with a common multiplicative constant.

Experimental tests of the riskless additive utility model, which rely only on the ordinal characteristics of the data, have been carried out by Adams and Fagot (1959), Delbeke and Fauville (1974), Fischer (1976), and Tversky (1967).

Adams and Fagot (1959) had 24 subjects make pairwise choices between hypothetical job applicants, each of which was characterized by his intelligence and ability to handle people. Each factor assumed four levels, resulting in 16 stimuli in total. For 80 percent of the subjects, at least 74 of the 77 choices were consistent with the additive utility model. Violations were largely due to intransitivities.

In a study by Tversky (1967), prisoners indicated their lowest selling prices for 16 combinations of one to four packs of cigarettes with one to four bags of candy. Each commodity bundle was judged three times and data analyses were performed on the median prices. Nine of the 11 subjects satisfied the additive utility model perfectly. The other two subjects showed only minor violations, the tau correlations between the model and their data being 0.983 and 0.966.

In the first of a series of experiments conducted by Delbeke and Fauville (1974), 10 high school boys were asked to rate the utility of 16 career by salary combinations as well as of 16 destination by transport mode combinations. Axiom and scaling analyses revealed that all but two data matrices satisfied the additive model very well. In a second experiment 10 high school girls rated 16 transport mode by company combinations and 16 apparatus by accessories combinations. Whereas for the first stimulus set the data of most subjects could be represented quite well by an additive model, only one subject satisfied additivity for the second stimulus set. The attribute levels for the second stimulus set were phonograph, tape recorder, camera, and slide projector for the apparatus factor, and ten LP's, five tapes, ten films, and four dozens of slides for the accessories factor. There clearly exist interdependencies between the two factors, which gave rise to many violations of the preference independence condition.

In a last experiment to be mentioned, Fischer (1976) had 10 students evaluate 27 hypothetical job offers. Each job offer was described in terms of the city of employment, the salary, and the type of work. On the basis of a conjoint measurement analysis, the additive model could be rejected conclusively for only one subject. Some other subjects showed rather large violations of single factor independence. Since substantial violations only occurred with respect to the factor the subjects judged to be least important, these violations do not necessarily prove the inadequacy of the additive model.

Summarizing the experiments discussed above, we can conclude that generally speaking substantial support was found for the riskless additive utility model. Violations occurred when the attributes were not psychologically independent of each other. In such a situation, it is sometimes possible to reformulate the decision problem in terms of independent attributes. Alternatively one may consider adopting an interactive model, such as a multiplicative or distributive model (cf. Krantz et al., 1971).

DECISIONS UNDER UNCERTAINTY

Since in most experimental tests of risky MAUT gambles are used as choice alternatives, I shall illustrate the various independence conditions in terms of simple gambles. Following von Winterfeldt and Fischer (1975), a gamble G with m mutually exclusive and exhaustive events E_j $(j = 1,m)$ will be represented as an $n \times m$ matrix whose rows refer to the attributes and whose columns indicate the events determining the values of each attribute. An example of such a matrix is presented in Figure 1.

All MAUT models for representing decisions made under risk are based on the expected utility (EU) model which was first proposed by Bernoulli (1738/1954) and further elaborated by von Neumann and Morgenstern (1947). The model can be written as

$$P \succsim Q \text{ iff } \sum_{\mathbf{x}\in A} P(\mathbf{x})U(\mathbf{x}) \geqslant \sum_{\mathbf{x}\in A} Q(\mathbf{x})U(\mathbf{x}) \qquad (3)$$

where U is a real-valued utility function on A. It should be noted that although it is attractive to consider a DM's behavior as optimizing a certain criterion, other criteria than maximizing expected utility are equally plausible, such as, for instance, minimizing risk or maximizing (subjective) expected utility subject to certain constraints.

$$
\begin{array}{c c}
 & G \\
 & \begin{array}{cccccc} E_1 & E_2 & \cdot & \cdot & \cdot & E_m \\ p_1 & p_2 & \cdot & \cdot & \cdot & p_m \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ \cdot \\ \cdot \\ A_n \end{array} & \left[\begin{array}{cccccc} x_1 & y_1 & \cdot & \cdot & \cdot & z_1 \\ x_2 & y_2 & & & & z_2 \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ x_n & y_n & & & & z_n \end{array}\right]
\end{array}
$$

FIG. 1. AN ILLUSTRATION OF A GAMBLE WITH MULTIDIMENSIONAL OUTCOMES

The critical assumptions on which (3) is based are transitivity and event independence.

DEFINITION 2. $(\pi, \dot{\succsim})$ satisfies *event independence* iff, for all $P, Q, R \times \pi$, $P \dot{\succsim} Q$ implies $aP+(1-a)R \dot{\succsim} aQ+(1-a)R$ for $0 < a < 1$.

Together with an Archimedean axiom event independence is sufficient to ensure a representation according to (3) given that $\dot{\succsim}$ is a weak order. The utility function U can be shown to be unique up to a linear transformation. An example of event independence is presented in Figure 2. Suppose you prefer G_1 to G_2, then event independence requires you to prefer also G_1^* to G_2^*.

$$
\begin{array}{cc}
G_1 & \\
\begin{array}{cc} E_1 & E_2 \\ p & (1-p) \\ [x & y] \end{array} & \dot{\succsim}
\end{array}
\begin{array}{c}
G_2 \\
\begin{array}{cc} E_1 & E_2 \\ q & (1-q) \\ [z & w] \end{array}
\end{array}
$$

iff

$$
\begin{array}{c}
G_1^* \\
\begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \\ \alpha p & \alpha(1-p) & (1-\alpha)r & (1-\alpha)(1-r) \\ [x & y & a & b] \end{array}
\end{array}
\dot{\succsim}
\begin{array}{c}
G_2^* \\
\begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \\ \alpha q & \alpha(1-q) & (1-\alpha)r & (1-\alpha)(1-r) \\ [z & w & a & b] \end{array}
\end{array}
$$

FIG. 2. AN ILLUSTRATION OF EVENT INDEPENDENCE

Just like in the riskless case $U(\mathbf{x})$ can be written as a function of the separate attribute utilities provided certain additional independence conditions hold. The three models for decomposing $U(\mathbf{x})$, that will be discussed in this paper, require some degree of utility independence (Keeney, 1971). Utility independence can be considered as the risky analog of preference independence. It is sometimes called strong conditional utility independence.

DEFINITION 3. A_I is *utility independent* of $A_{\bar{I}}$ iff, for all $P, Q \in \pi_I$,

$(P_I,\mathbf{a}_{\bar{I}}) \mathrel{\dot{\geqslant}} (Q_I,\mathbf{a}_{\bar{I}})$ for some $\mathbf{a}_{\bar{I}} \in A_{\bar{I}}$, implies $(P_I,\mathbf{b}_{\bar{I}}) \mathrel{\dot{\geqslant}} (Q_I,\mathbf{b}_{\bar{I}})$ for every $\mathbf{b}_{\bar{I}} \in A_{\bar{I}}$.

Figure 3 gives an illustration of utility independence. If you prefer G_1 to G_2, then you should also prefer G_1^* to G_2^* in order for $A_{\{1,2\}}$ to be utility independent of $A_{\{3,4,5\}}$. Regardless of your choice between G_1 and G_2, and regardless of the event that occurs, you will obtain $\mathbf{a}_{\{3,4,5\}}$. Therefore $\mathbf{a}_{\{3,4,5\}}$ is called a "sure thing".

$$\begin{array}{cc} \multicolumn{2}{c}{G_1} \\ E_1 & E_2 \\ p & (1-p) \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{G_2} \\ E_1 & E_2 \\ q & (1-q) \end{array}$$

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ a_3 & a_3 \\ a_4 & a_4 \\ a_5 & a_5 \end{bmatrix} \mathrel{\dot{\geqslant}} \begin{bmatrix} z_1 & w_1 \\ z_2 & w_2 \\ a_3 & a_3 \\ a_4 & a_4 \\ a_5 & a_5 \end{bmatrix}$$

iff

$$\begin{array}{cc} \multicolumn{2}{c}{G_1^*} \\ E_1 & E_2 \\ p & (1-p) \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{G_2^*} \\ E_1 & E_2 \\ q & (1-q) \end{array}$$

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ b_3 & b_3 \\ b_4 & b_4 \\ b_5 & b_5 \end{bmatrix} \mathrel{\dot{\geqslant}} \begin{bmatrix} z_1 & w_1 \\ z_2 & w_2 \\ b_3 & b_3 \\ b_4 & b_4 \\ b_5 & b_5 \end{bmatrix}$$

FIG. 3. AN ILLUSTRATION OF UTILITY INDEPENDENCE

If A_I is utility independent of $A_{\bar{I}}$ for every $I \subset N$, $I \neq \varnothing$, then marginal independence (cf. Fishburn, 1968, pp. 358-359) should be tested.

DEFINITION 4. $A_1, A_2, \ldots, A_n$ satisfy *marginal independence* iff, for all $P, Q \in \pi$, $P_{\{k\}} = Q_{\{k\}}$ for every $k \in N$ implies $P \doteq Q$.

Marginal independence states that two alternatives are judged to be equivalent whenever they have the same marginal probability distributions on each attribute. If G is a gamble with equally likely events, then marginal independence requires you to be indifferent between G and any other gamble whose matrix is identical to G except for a permutation within each row. An example is given in Figure 4.

$$\begin{array}{ccc} \multicolumn{3}{c}{G} \\ E_1 & E_2 & E_3 \\ 1/3 & 1/3 & 1/3 \end{array} \qquad \begin{array}{ccc} \multicolumn{3}{c}{G^*} \\ E_1 & E_2 & E_3 \\ 1/3 & 1/3 & 1/3 \end{array}$$

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \\ x_4 & y_4 & z_4 \end{bmatrix} \doteq \begin{bmatrix} x_1 & z_1 & y_1 \\ y_2 & x_2 & z_2 \\ z_3 & x_3 & y_3 \\ y_4 & z_4 & x_4 \end{bmatrix}$$

FIG. 4. AN ILLUSTRATION OF MARGINAL INDEPENDENCE

If A_I is utility independent of $A_{\bar{I}}$ for every $I \subset N$, $I \neq \varnothing$, and if marginal independence holds, $U(\mathbf{x})$ can be decomposed additively:

$$U(\mathbf{x}) = \sum_{i \in N} u_i(x_i). \tag{4}$$

When combined with (3), (4) defines the additive EU model proposed by Fishburn (e.g., 1968, 1970). If marginal independence is violated but A_I is utility independent of $A_{\bar{I}}$ for every $I \subset N$, $I \neq \varnothing$, a representation according to Keeney's (1974) multiplicative EU model is possible:

$$1 + kU(\mathbf{x}) = \prod_{i \in N} [1 + ku_i(x_i)] \ , \ k \neq 0. \tag{5}$$

For $n = 3$, eq. (5) becomes

$$U(\mathbf{x}) = u_1(x_1) + u_2(x_2) + u_3(x_3) + ku_1(x_1)u_2(x_2) + ku_1(x_1)u_3(x_3) + ku_2(x_2)u_3(x_3) + k^2u_1(x_1)u_2(x_2)u_3(x_3) \ .$$

It is clear form the above equation that the multiplicative EU model allows for certain types of interactions between the separate utility scales. If A_I is only independent of $A_{\bar{I}}$ for $I = \{k\}$, $k \in N$, then the quasi-additive EU model is indicated (Keeney, 1972):

$$U(\mathbf{x}) = \sum_{I \subset N} c_I \left[\prod_{i \in I} u_i(x_i) \right]. \tag{6}$$

Expanding (6) for $n = 3$ yields

$$U(\mathbf{x}) = c_1u_1(x_1) + c_2u_2(x_2) + c_3u_3(x_3) + c_{12}u_1(x_1)u_2(x_2) + c_{13}u_1(x_1)u_3(x_3) + c_{23}u_2(x_2)u_3(x_3) + c_{123}u_1(x_1)u_2(x_2)u_3(x_3) \quad ,$$

which shows that the quasi-additive EU model can capture more kinds of interdependencies among the attribute utilities than the multiplicative EU model.

Although the additive, multiplicative and quasi-additive EU models have been widely used in applied research, very few studies have concentrated on testing the descriptive validity of the underlying independence assumptions.

The previously mentioned study by Delbeke and Fauville (1974) was explicitly designed to test the marginal independence axiom. Subjects are presented pairs of gambles which had equal marginal probability distributions on each attribute. The outcomes of the gambles were the same stimuli that were used before for validating the riskless additive utility model (cf. supra). A subject was considered indifferent between the two gambles of a pair if both were chosen approximately equally often over a number of replications (indifference judgments were not allowed). In general marginal independence was not supported very well, even when the subject's riskless choices were clearly additive.

In the second part of an experiment conducted by Fischer (1976), subjects had to adjust the probabilities of a series of two-event gambles such that the subjects would be indifferent between the gamble and a sure alternative. The outcomes were the same job offers used in the first part of the experiment, which I discussed in the previous section. Assuming the validity of (3), Fischer obtained an estimate of the overall utility of each job offer on the basis of those probability judgments. These estimates were subsequently subjected to a conjoint measurement analysis. The utility estimates of three of the 10 subjects turned out to be clearly non-additive. Unfortunately no attempt was made to determine whether this was due to violations of (4) or to the inadequacy of (3). A multiple regression on the overall utility estimates using single attribute utility ratings as independent variables revealed that for six of the ten subjects the goodness of fit was significantly increased by including interaction terms. However this analysis is based on the assumption that the single attribute utility ratings form an interval scale.

A direct test of event, utility and marginal independence was attempted by von Winterfeldt (1980). The test items were pairs of fifty-fifty gambles with outcomes consisting of a certain amount of gasoline combined with a certain amount of ground beef. The most interesting results were obtained in the tests for marginal independence, which were based on the presumed natural indifference between G_1 and G_2 in Figure 5 when both *a* and *b* are zero. Marginal independence

$$
\begin{array}{lcc}
 & G_1 & G_2 \\
 & \begin{array}{cc} E_1 & E_2 \\ {}^1/_2 & {}^1/_2 \end{array} & \begin{array}{cc} E_1 & E_2 \\ {}^1/_2 & {}^1/_2 \end{array} \\
\begin{array}{l} \text{GAS} \\ \text{BEEF} \end{array} & \begin{bmatrix} 16 & 0 \\ a & b \end{bmatrix} & \begin{bmatrix} 0 & 16 \\ a & b \end{bmatrix}
\end{array}
$$

FIG. 5. TEST ITEMS USED BY VON WINTERFELDT (1980) FOR TESTING MARGINAL INDEPENDENCE

requires the subject to be also indifferent between the gambles when *a* and *b* assume non-zero values. However the subjects showed a consistent tendency to prefer the gamble whose outcomes were more equally distributed across the two events. This is a clear demonstration of what has been called multivariate risk aversion (Richard, 1975). Multivariate risk aversion is incompatible with the additive EU model, but not with the multiplicative and quasi-additive EU models. The other two independence axioms were tested in a similar way, except that instead of starting off with a pair of gambles between which the subject was naturally indifferent, an indifference pair was constructed by having the subjects adjust the values of one dimension of one of the two gambles. Interpretation of the results of those tests was greatly hampered by the strong bias subjects showed to choose the gamble with the matched value. No convincing evidence was obtained that event or utility independence were violated other than through that bias.

Whereas some years ago Fischer (1975) concluded that "the little data available suggest that additive utility models provide an excellent approximation to preference under risk" (p. 19), the experiments discussed above rather suggest that the marginality assumption, which is crucial for the additive EU model, is systematically violated, especially by risk aversiveness. More empirical tests of event and utility independence are needed before the adequacy of the multiplicative and quasi-additive EU models can be evaluated.

DISCUSSION

In the previous sections, the most popular multi-attribute utility models for representing riskless and risky choices were reviewed, emhasizing the independence assumptions on which they are based. If one is merely interested in MAUT as a prescriptive theory, testing these assumptions is probably not that important. However if one is concerned with the descriptive validity of MAUT, these axioms should be taken very seriously. As I stressed before, much can be learned about the psychology of the underlying judgmental processes from a careful test of these axioms. Since all MAUT models are compensatory in nature, it is obvious that they will not be adequate in every situation. Therefore, the real question to be asked is not whether a certain axiom is satisfied or not, but under what conditions an axiom holds. Hopefully future psychological research on MAUT will be performed within that perspective.

REFERENCES

ADAMS, E., & FAGOT, R. A model of riskless choice. *Behavioral Science*, 1959, *3*, 1-10.

BERNOULLI, D. Specimen theoriae novae de mensura sortis. *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, 1738, *5*, 175-192. (Translated in English by L. SOMMER, *Econometrica*, 1954, *22*, 23-36.)

DELBEKE, L., & FAUVILLE, J. An empirical test for Fishburn's additivity axiom. *Acta Psychologica*, 1974, *38*, 1-20.

FARQUHAR, P.H. A survey of multiattribute utility theory and applications. In M.K. STARR & M. WELENY (Eds.), *Multiple criteria decision making. TIMS Studies in the Management Sciences*, 1977, *6*, 59-89.

FARQUHAR, P.H. Interdependent criteria in utility analysis. In S. ZIONTS (Ed.), *Multiple criteria problem solving*. Berlin: Springer-Verlag, 1978.

FISCHER, G.W. Experimental applications of multi-attribute utility models. In D. WENDT & C. VLEK (Eds.), *Utility, probability, and human decision making*. Dordrecht: Reidel, 1975.

FISCHER, G.W. Multidimensional utility models for risky and riskless choice. *Organizational Behavior and Human Performance*, 1976, *17*, 127-146.

FISHBURN, P.C. Utility theory. *Management Science*, 1968, *14*, 335-378.

FISHBURN, P.C. *Utility theory for decision making*. New York: Wiley, 1970.

KEENEY, R.L. Utility independence and preferences for multiattributed consequences. *Operations Research*, 1971, *19*, 875-893.

KEENEY, R.L. Utility functions for multiattributed consequences. *Management Science*, 1972, *18*, 276-287.

KEENEY, R.L. Multiplicative utility functions. *Operations Research*, 1974, *22*, 22-34.

KEENEY, R.L., & RAIFFA, H. *Decisions with multiple alternatives: preferences and value tradeoffs*. New York: Wiley, 1976.

KRANTZ, D.H., LUCE, R.D., SUPPES, P., & TVERSKY, A. *Foundations of measurement* (Vol. 1). New York: Academic Press, 1971.

RICHARD, S.F. Multivariate risk aversion, utility independence and separable utility functions. *Management Science*, 1975, *22*, 12-21.

TVERSKY, A. Additivity, utility and subjective probability. *Journal of Mathematical Psychology*, 1967, *4*, 175-201.

TVERSKY, A. Intransitivity of preferences. *Psychological Review*, 1969, *76*, 31-48.

von NEUMANN, J., & MORGENSTERN, O. *Theory of games and economic behavior*. Princeton: Princeton University Press, 1947.

von WINTERFELDT, D. Additivity and expected utility in risky multi-attributed preferences. *Journal of Mathematical Psychology*, 1980, *21*, 66-82.

von WINTERFELDT, D., & FISCHER, G.W. Multi-attribute utility theory: models and assessment procedures. In D. WENDT & C. VLEK (Eds.), *Utility, probability, and human decision making*. Dordrecht: Reidel, 1975.

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